LEVEL II (12) SC

TECHNICAL REPORT T-78-61

RADAR TRACK DATA CORRELATION OR REACHABLE SETS REVISITED: THE REACHABLE STATE

U.S. ARMY
MISSILE
RESEARCH
AND
DEVELOPMENT
COMMAND

Richard E. Dickson
Systems Simulation Directorate
Technology Laboratory

31 July 1978

Redstone Arsenal, Alabama 35809

Approved for public release; distribution unlimited.

DMI FORM 1000, 1 APR 77

DISPOSITION INSTRUCTIONS

DESTROY THIS REPORT WHEN IT IS NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.

DISCLAIMER

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

TRADE NAMES

USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL INDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER T-78-61	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Radar Track Data Ccrrelation or Reachable Sets Revisited: The Reachable State.		5. TYPE OF REPORT & PERIOD COVERED (9) Technical Report.
		6. PERFORMING ORG. REPORT NUMBER (14) DRDMI-T-78-61
7. AUTHOR(s) (10) Richard E. Dickson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Commander US Army Missile Research and Development Command ATTN: DRDMI-TD Redstone Arsenal, Alabama 35809		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Commander US Army Missile Research and Development Command ATTN: DRDMI-TI Redstone Arsenal, Alabama 35809		12. REPORT DATE ~~29 June 1978~~ 13. NUMBER OF PAGES 28
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Cleared for Public Release;
Unlimited Distribution

(11) 31 Jul 78 (12) 34p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aircraft Tracking
Radar Track Data Correlation
Reachable Sets
Air Traffic Control

sub ϕ

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Given an aircraft's position and velocity at time t_o and the limits on the rate at which the aircraft executes horizontal turns, what is the set of all possible position at time t, given the change in heading at time t and that the speed is unc[illegible]

79 02 27 005

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED LB

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

393 427

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SYMBOLS

a	Acceleration normal to the velocity vector.
A	Initial Position. For a given change in heading, theta
C	The point in the reachable set reached by a continuous turn.
I	The point on the inner boundary reached by a turn at the maximum rate.
O	The point on the outer boundary reached by a turn at the maximum rate.
r	Radius of turn for the maximum rate of turn (Equation (5)).
t	Time.
t_o	Initial time.
Δt, T	Time increment or delay.
T_1	Time spent in turning, $o \leq T_1 \leq T$.
v	Speed, magnitude of the velocity.

Position relative to the initial position

X	Along
Y	Normal to the linearly extrapolated flight path.

Position relative to the linearly extrapolated position

x	Along
y	Normal to the linearly extrapolated flight path.

For a given change in heading the slope of the line containing

α	All points on all possible inner boundaries.
β	All points on all possible outer boundaries.
γ	All points between the inner outer boundaries of the reachable set.
θ	Change in heading or course.
$\dot{\theta}$	Rate of change in heading, rate of turn (Equation (6)).

SUBSCRIPTS

$(\ )_o$	Outer boundary.
$(\ )_i$	Inner boundary.
$(\ )_c$	Continuous turn.
$(\ )_{max}$	Maximum.
$(\ )_{min}$	Minimum.

ACKNOWLEDGEMENT

The author was referred to the paper on reachable sets [1] by Joe Leva (and an unknown person) at MITRE after the author had presented an approach which is essentially a special case of the reachable set, the continuous turn.

Graphics support (and error correction) was (were) furnished by Clarence Wood, DMIS, US Army Missile Readiness Command, Redstone Arsenal.

1. INTRODUCTION

In the 1980's, tactical air (traffic) control and air defense systems will exchange air track data via digital communication links. Where radar coverages overlap, the forwarding delays in the communication links may cause problems in correlating locally held air track data with that forwarded from a remote reporting unit. One is reminded of Segal's Law: *"A man with one watch knows what time it is. A man with two watches is never sure."*

A point which should be made explicit is that the problem is one of correlation of processed data; that is, the data has already been filtered. If one wished to filter again, it would be necessary to know the filters which had already been used since they would become a part of the plant or observation matrix. Such an approach does not appear practical. Since the radar track data are estimates, it should be possible to treat them in a deterministic fashion.

If the only "maneuver" one could expect was no maneuver, there would be "little" difficulty in correlating tracks, but, unfortunately, this is not the case. Though not the only maneuver one would expect, no maneuver is probably the most likely "maneuver" since most of an aircraft's flight time would be spent in level flight at constant speed and heading. Of course, this implies the aircraft is unalerted or cooperative.

The next most likely maneuver would seem to be a change in course or heading, that is, a single turn in the horizontal plane.

The above maneuvers are rather "simple" and are those one would expect of commercial aircraft or tactical aircraft in formation. Tactical aircraft would additionally execute maneuvers such as the split "S", Immelman turns, loops, diving spirals, breaks, diving turns and the like when conditions warrant.

Hoares' Law of Large Problems: *"Inside every large problem is a small problem struggling to get out."* In the following, the emphasis will be on single horizontal turns at constant speed. As in Reference 2, the approach will be geometric "because it provides more insight than other alternatives," and, also, at times heuristic.

2. REACHABLE SETS REVISITED

In Reference 1 and 2 the question, "Given the aircraft position and velocity at time $t - \Delta t$ and the limits on the ability of the aircraft to maneuver and/or change speed, what is the set of

all possible aircraft positions at time t?" is addressed. The set of all possible positions was called the reachable set (or set of attainability) and was described by a set of equations for the inner and outer boundaries. Only the assumptions and results are presented here since the derivation is readily available [1].

The following assumptions were made concerning the motion of the aircraft [1]:

- *"Any lateral maneuver in the xy-plane is due to a constant-rate turn at constant speed.*
- *Only one turn maneuver is allowed during the time increment Δt.*
- *The turning maneuver can begin or end at any time during the time increment Δt.*
- *Speed variations and turning maneuvers are assumed to be independent of each other.*
- *Any speed change is assumed to take place at the beginning of the time increment Δt, and the speed is assumed to be constant throughout this time increment."*

The outer boundary of the reachable set is determined by initially turning to a new heading and then flying out along that heading. This maximizes the departure from the initial position along a given heading relative to the initial position [1]. Of course, flight along the initial heading maximizes the departure from the initial position. The inner boundary for a single turn is determined by flying along the initial heading till the last possible moment before executing a change in heading; that is, the turn is at the end rather than the beginning.

The outer boundary of the reachable set is given by [1].

$$X_o = r \sin \theta + (v \cdot \Delta t - r \cdot \theta) \cos \theta \quad (1)$$

$$Y_o = \pm [r (1 - \cos \theta) + (v \cdot \Delta t - r \cdot \theta) \sin \theta] \quad (2)$$

and the inner boundary by [1].

$$X_i = r \sin \theta + (v \cdot \Delta t - r \cdot \theta) \quad (3)$$

$$Y_i = \pm r (1 - \cos \theta) \quad (4)$$

Since

$$r = \frac{v^2}{a} \quad (5)$$

and

$$\dot{\theta} = \frac{a}{v} \quad (6)$$

The above equations, (1), (2), (3) and (4), may be rewritten

$$X_o = \frac{v^2}{a} \sin\left(\frac{aT_1}{v}\right) + v(T-T_1) \cos\left(\frac{aT_1}{v}\right) \quad (7)$$

$$Y_o = \pm\left[\frac{v^2}{a}\left(1 - \cos\left(\frac{aT_1}{v}\right)\right) + v\,(T-T_1)\,\sin\left(\frac{aT_1}{v}\right)\right] \quad (8)$$

and

$$X_i = \frac{v^2}{a}\sin\left(\frac{aT_1}{v}\right) + v\,(T-T_1) \quad (9)$$

$$Y_i = \pm\left[\frac{v^2}{a}\left(1 - \cos\left(\frac{aT_1}{v}\right)\right)\right] \quad (10)$$

It should be noted that the inner and outer boundaries coincide at two points, when $T_1 = T$ and $T_1 = 0$.

Since the most likely maneuver may be "no maneuver", it would seem reasonable to relocate the origin at the linearly extrapolated position of the aircraft.

Relative to the linearly extrapolated position, the outer and inner boundaries are

$$x_o = \frac{v^2}{a}\sin\left(\frac{aT_1}{v}\right) + vT\left(\cos\left(\frac{aT_1}{v}\right) - 1\right) - vT_1\cos\left(\frac{aT_1}{v}\right) \quad (11)$$

$$y_o = \pm\left[\frac{v^2}{a}\left(1 - \cos\left(\frac{aT_1}{v}\right)\right) + v\,(T-T_1)\,\sin\left(\frac{aT_1}{v}\right)\right] \quad (12)$$

and

$$x_i = \frac{v^2}{a}\sin\left(\frac{aT_1}{v}\right) - vT_1 \quad (13)$$

$$y_i = \pm\left[\frac{v^2}{a}\left(1 - \cos\left(\frac{aT_1}{v}\right)\right)\right] \quad (14)$$

respectively. The inner boundary is no longer a function of the total delay, T, but only of the time spent in turning, T_1, that is, the inner boundary coincide for all time delays.

Perhaps a gedanken experiment will help clarify this point. Consider a string of length vT and a circle whose radius is the radius of turn,

$$r = \frac{v^2}{a} \quad (15)$$

Let the string be attached to the circle and the point of attachment be the tangent point between the circle and the linearly extrapolated flight path (see *Figure 1*).

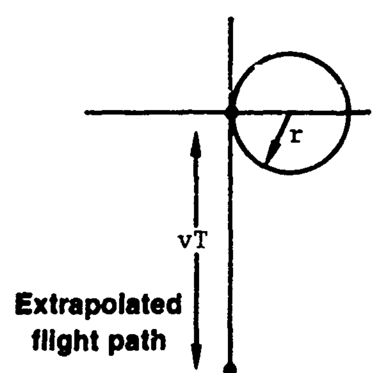

Figure 1.

The other end of the string is attached to the initial position of the aircraft, that is, the position at time t_0. As the circle rolls (without slipping) back along the extrapolated flight path, the string is wound up (without stretching) on the circle. The locus of

the point of the string's attachment to the circle describes a cycloid [3], the inner boundary of the reachable set. When the string is all wound up, the circle will stop rolling and will be tangent to the linearly extrapolated flight path at the aircraft's initial position. The other half of the inner boundary would be constructed in a similar fashion on the other side of the linearly extrapolated flight path.

Now disconnect the string from the circle, holding the circle fixed and keeping tension on the string, unwind the string from the circle. The involute† generated by the end of the string is the outer boundary of the reachable set [2].

If, for some reason, one knew the linearly extrapolated position but not the time delay, one could construct the inner boundary, but the outer boundary and where it met the inner boundary would not be known.

Since

$$\dot{\theta} = \frac{a}{v} \tag{16}$$

The equations for the inner and outer boundaries may be written

$$\frac{x_o \dot{\theta}_{max}}{v} = \sin\theta + (\theta_{max} - \theta)\cos\theta - \theta_{max} \tag{17}$$

$$\frac{y_o \dot{\theta}_{max}}{v} = (1 - \cos\theta) + (\theta_{max} - \theta)\sin\theta \tag{18}$$

and

$$\frac{x_i \dot{\theta}_{max}}{v} = \sin\theta - \theta \tag{19}$$

$$\frac{y_i \dot{\theta}_{max}}{v} = (1 - \cos\theta) \tag{20}$$

respectively, where

$$\theta_{max} = \dot{\theta}_{max} T , \tag{21}$$

and

$$\theta = \dot{\theta}_{max} T_1 . \tag{22}$$

Figure 2 is a dimensionless plot of the reachable set; actually the dimensions are radians. It shows the common inner boundary and various outer boundaries. In *Figure 3* the total time delays for the outer boundaries were computed using Equation (21) and a standard rate of turn which is three degrees per second. The axes now also have the dimensions of time. In *Figure 4* the speed was chosen to be 300 meters per second (mach 0.91, 583 knots). One multiplies the speed by the times on the axes to obtain the displacements. Of course, one may choose any rate of turn, and give a rate of turn, any speed.

---

†whose evolute is the circle [3].

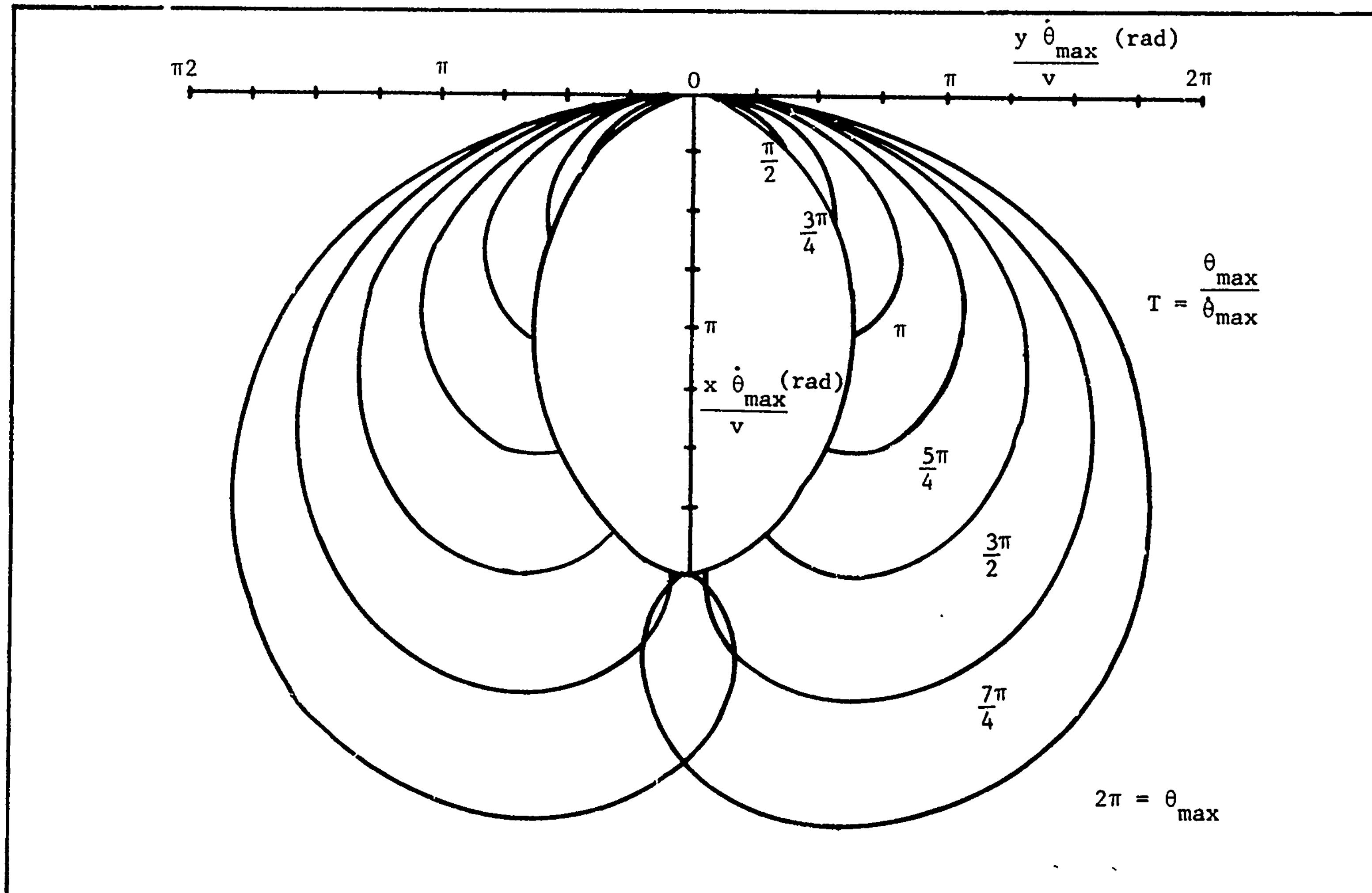

Figure 2. Dimensionless plot of inner and outer boundaries relative to the linearly extrapolated position.

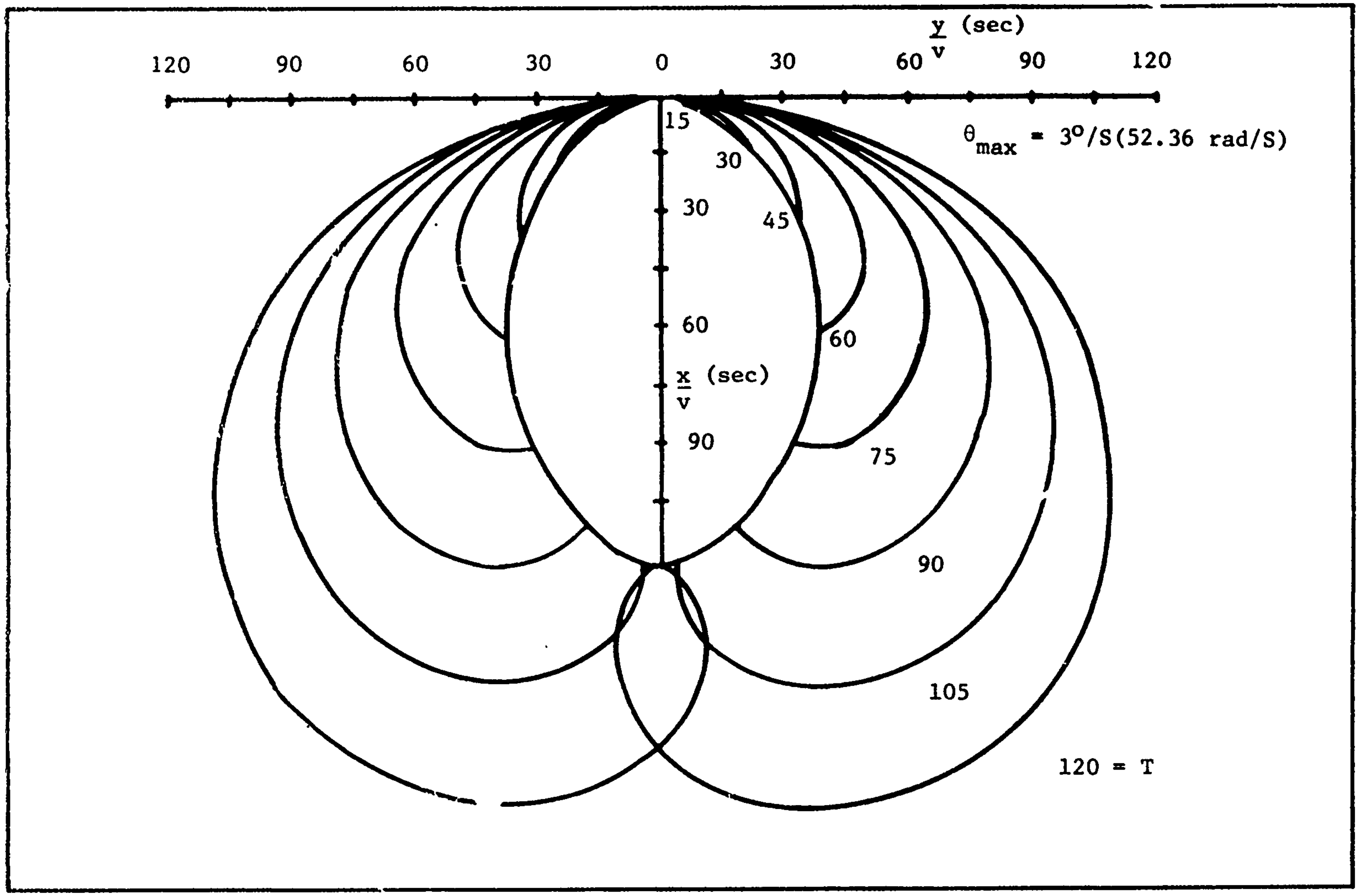

Figure 3. Reachable set for a standard turn.

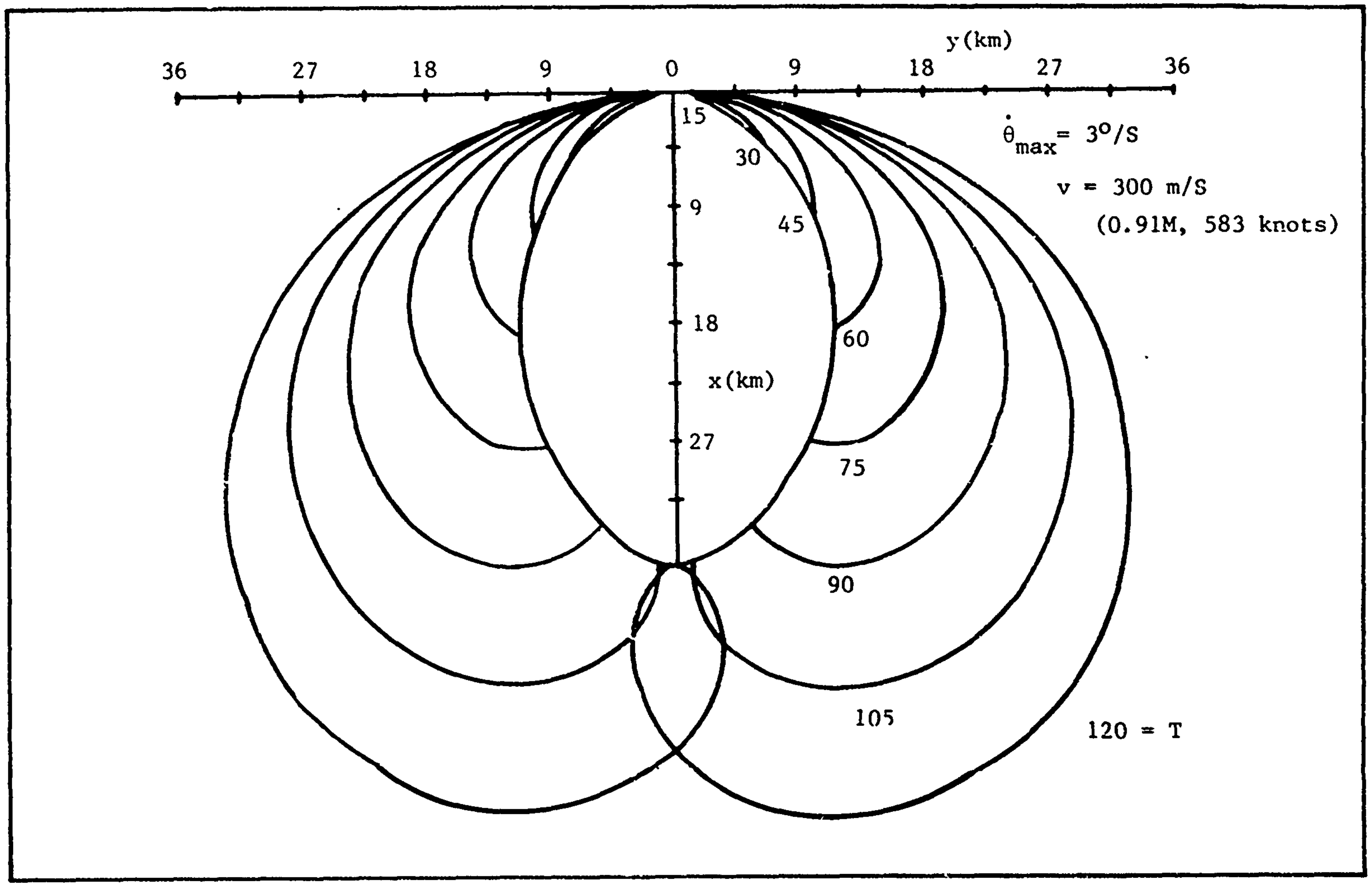

Figure 4. Reachable set for a standard turn and a speed of 300 m/s. milliseconds.

To construct the reachable set, it was necessary to know the velocity (speed and heading) and position at the initial time, t_0. If, in addition to the position, the velocity is known at time t, how might this additional information be used? Obviously one would compare the speeds to determine if they were equal, and, if equal, is the change in heading, if any, within the capabilities of the aircraft?

3. THE REACHABLE STATE

Given the aircraft's position, speed and heading at time t_0 and the limits on the rate at which the aircraft executes horizontal turns and that the speed is unchanged, what is the set of all possible positions at time t given the heading at time t?

Before the turn

$$\dot{x} = v \quad (23)$$

$$\dot{y} = o \quad (24)$$

but after a single turn

$$\dot{x} = v \cos \theta \quad (25)$$

$$\dot{y} = v \sin \theta \quad (26)$$

and, it follows that,

$$\theta = \arctan (\dot{y}/\dot{x}) \quad (27)$$

which is not unexpected.

What is the slope, γ, of the line connecting points on the inner and outer boundaries for a single turn that have the same change in heading?

$$\tan \gamma = \frac{x_o - x_i}{y_o - y_i} \quad (28)$$

$$= \frac{\cos \theta - 1}{\sin \theta} \quad (29)$$

$$= - \tan (\theta/2) \quad (30)$$

Therefore, the magnitude of the slope γ is half the change in heading.

From another gedanken experiment it will become apparent that, for a given rate of turn, the change in heading is the same at all points along the slope.

Consider the case where the rate of turn is the maximum rate. As the circle which generated the involute slides (without rolling) up the extrapolated flight path, the end of the string moves linearly from the outer boundary to the inner boundary. Linearly since the amount of string on the (arc of the) circle remains constant and any string added between the fixed end (initial

position) and the circle along the extrapolated flight path must be lost between the circle and the free end.

The reachable set for any turning rate less than the maximum is a subset of (is contained in) the reachable set of interest (*Figure 5*). Each of the possible subsets would contain a similar line segment of slope γ between its inner and outer boundaries. The combination of all such line segments would define an area within which any point in the reachable set with a change in heading of theta, Θ, would lie. The line segment for the maximum turning rate discussed above defines one boundary of the area in question.

Consider the slope of points on the inner boundaries of all reachable subsets† for a given change in heading. Dividing Equation (13) by Equation (14)

$$\frac{x_i(\dot{\theta}_1) - x_i(\dot{\theta}_2)}{y_i(\dot{\theta}_1) - y_i(\dot{\theta}_2)} = \frac{\sin\theta - \theta}{1 - \cos\theta} \tag{31}$$

and taking the arctangent

$$\alpha = \arctan\left(\frac{\sin\theta - \theta}{1 - \cos\theta}\right) \tag{32}$$

gives the desired slope. Equation (32) does not appear very pleasant but a "small angle" approximation makes it less formable,

$$\alpha \simeq \frac{\left(\theta - \frac{\theta^3}{3!} + \ldots\right) - \theta}{1 - \left(1 - \frac{\theta^2}{2!} + \ldots\right)} \tag{33}$$

$$\simeq \frac{\theta}{3} \tag{34}$$

A check on the accuracy of the approximation for various changes in heading will be found in *Table 1*.

The "small angle" approximation is quite excellent, that is, "good enough for government work." Beyond 180° the approximation breaks down seriously. Tactical computers will be pressed for CPU time and memory and approximations can reduce the computational burden.

The slope γ is the slope of the line segment between points on the inner and outer boundary that have the same change in heading. The slope α is the slope of the line segment between the inner boundary and the point reached by a continuous turn whose change in heading is theta. The minimum turning rate for a change in heading theta is

$$\dot{\theta}_{min} = \frac{\theta}{T} \tag{35}$$

It follows from Equation (13) and (14) that the point reached by

† for all possible turning rates $\leqq \dot{\theta}$ max.

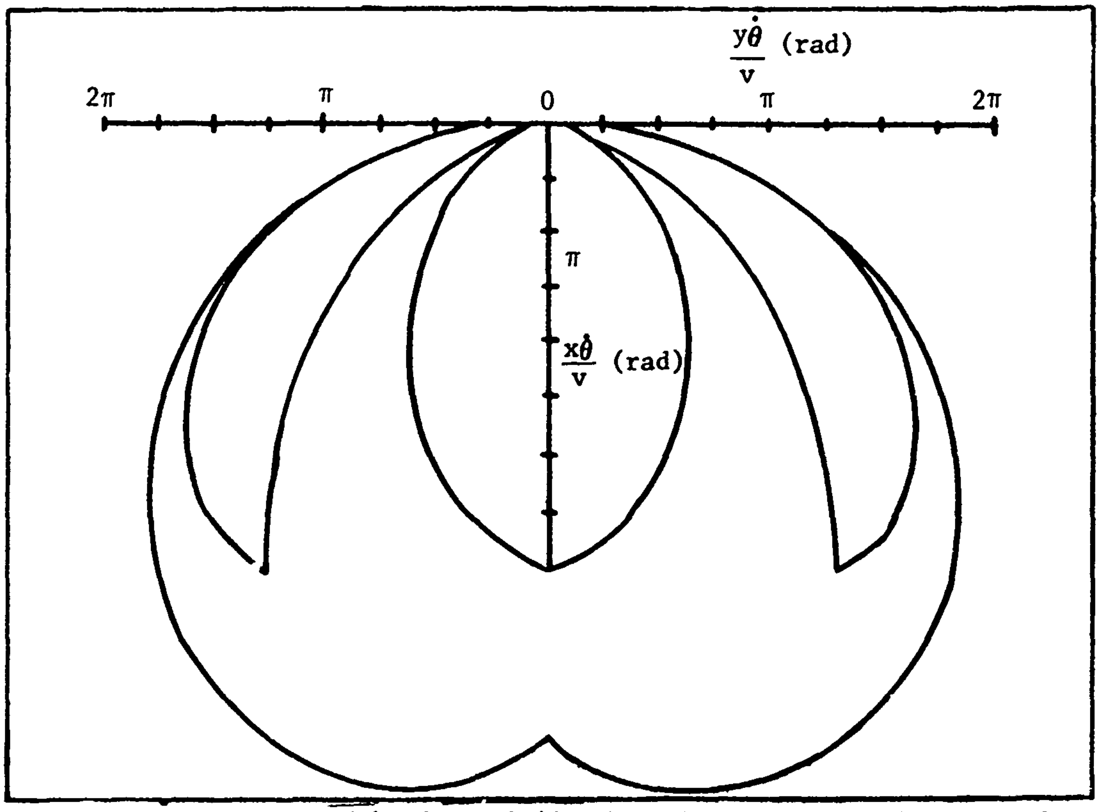

Figure 5. Reachable set containing the reachable subset whose rate of turn is half the maximum.

TABLE 1. COMPARISON OF EXACT AND APPROPRIATE SOLUTIONS OF ALPHA, α.

θ (deg)	$\arctan\left(\frac{\sin\theta - \theta}{1 - \cos\theta}\right)$ (deg)	$\frac{\theta}{3}$ (deg)	Error %
0	0.00	0	0.0
45	14.97	15	0.2
90	29.72	30	0.9
135	44.01	45	2.2
180	57.52	60	4.3
270	80.07	90	12.4
360	90.00	120	33.3

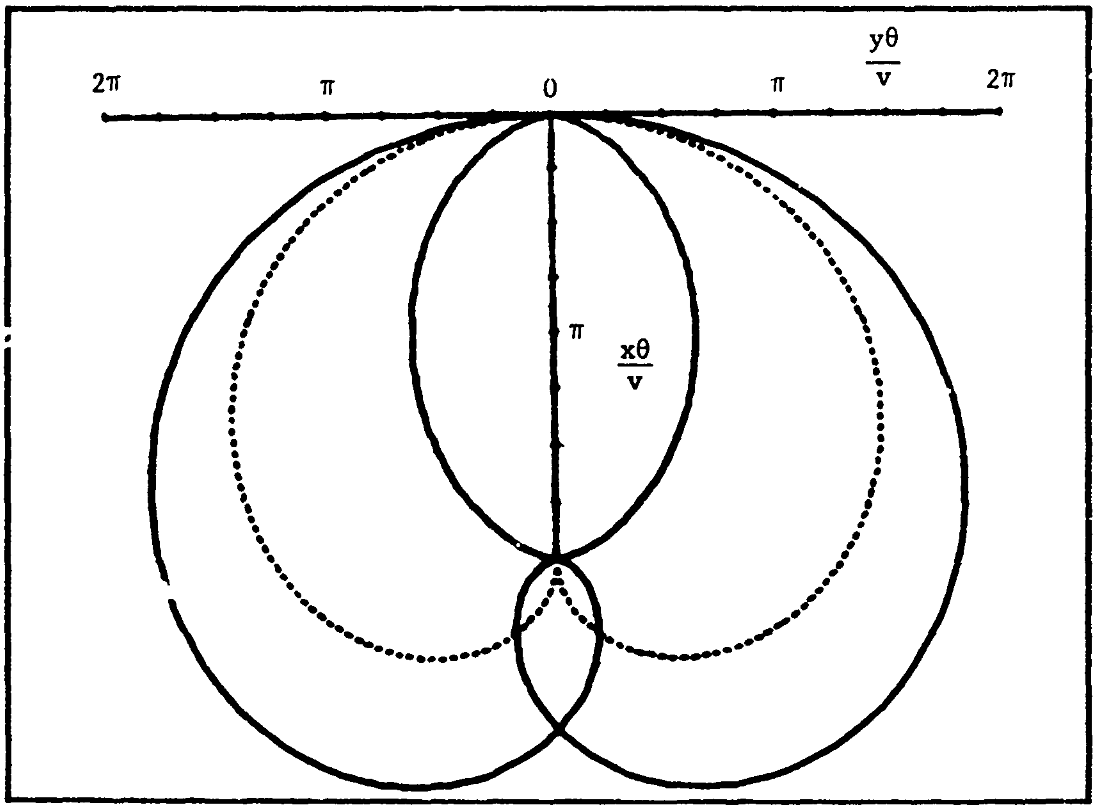

Figure 6. Reachable set for theta max of 2 π .

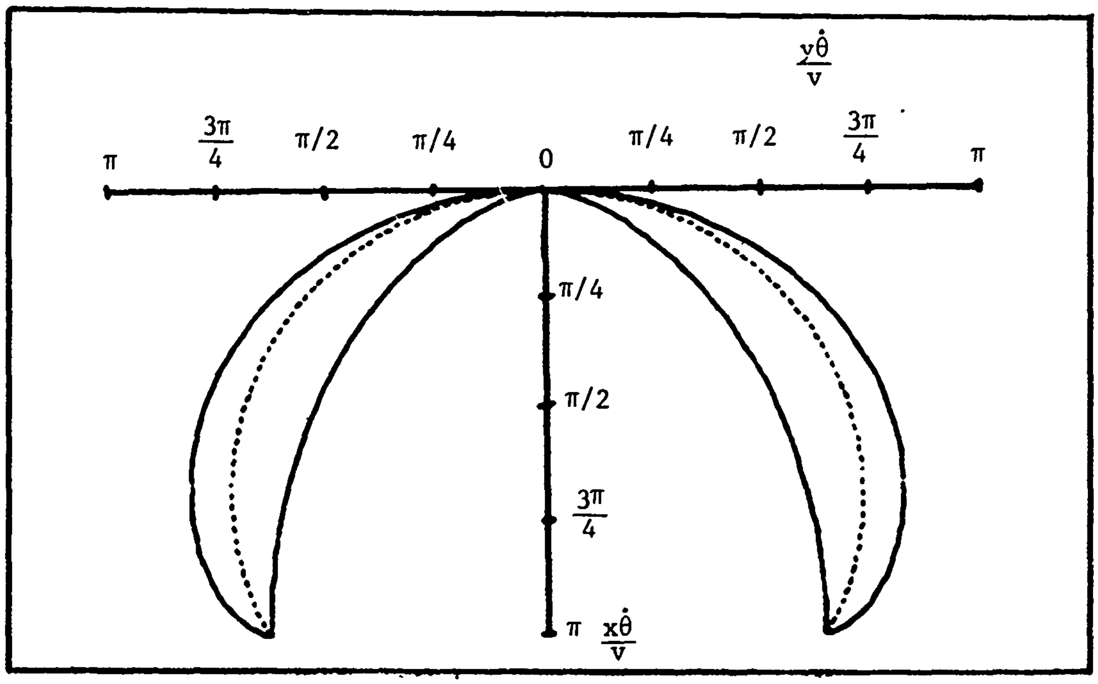

Figure 7. Reachable set for theta max of π.

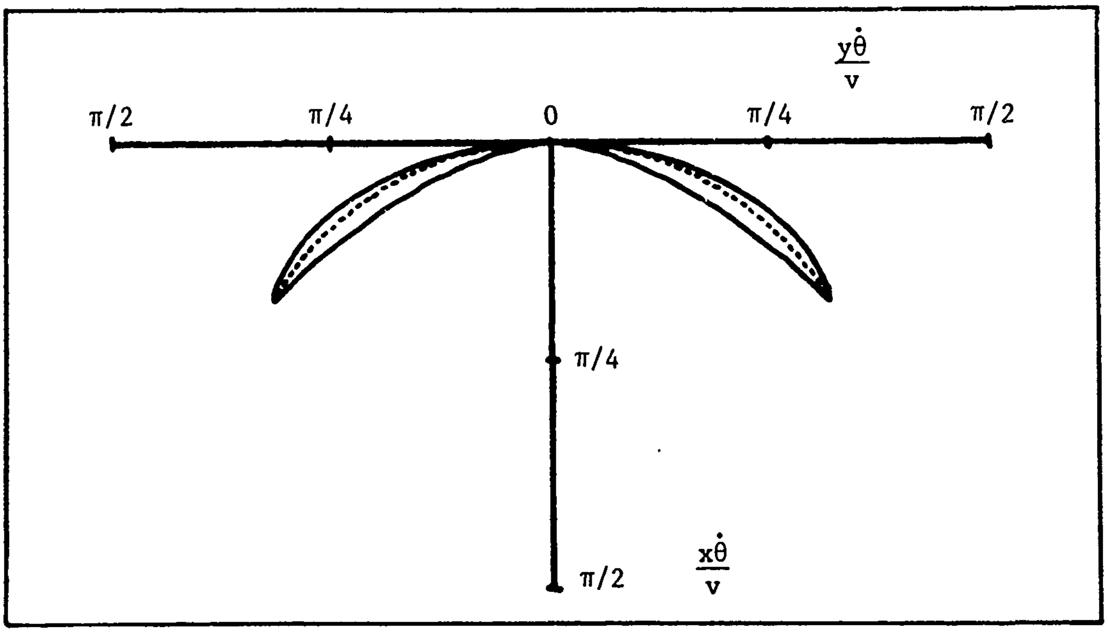

Figure 8. Reachable set for theta max of π/2.

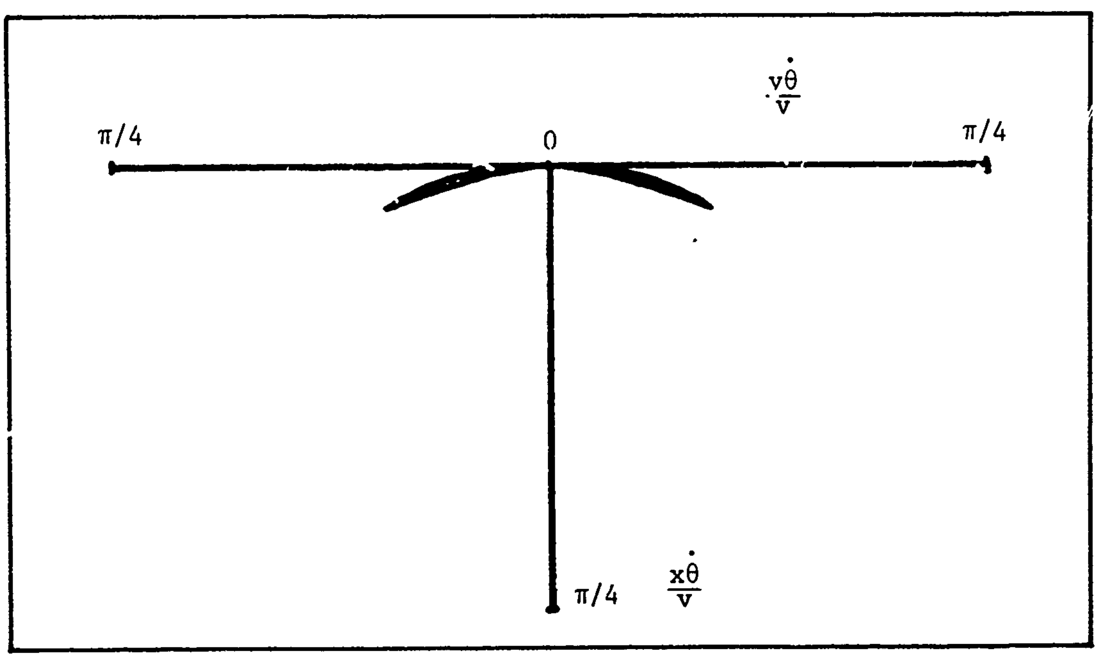

Figure 9. Reachable set for theta max of π/4.

continuously turning at the minimum turning rate is

$$x_c = \frac{vT}{\theta} (\sin \theta - \theta) \quad (36)$$

$$y_c = \frac{vT}{\theta} (1 - \cos \theta) \quad (37)$$

If nondimensional plots are desired,

$$\frac{x_c \dot{\theta}_{max}}{v} = \frac{\theta_{max}}{\theta} (\sin \theta - \theta) \quad (38)$$

and

$$\frac{y_c \dot{\theta}_{max}}{v} = \frac{\theta_{max}}{\theta} (1 - \cos \theta) \quad (39)$$

Figures 6 through *9* are nondimensional plots like *Figure 1* with "continuous turns" added. One would follow the same procedure used in *Figures 3* and *4* to intı ›duce the desired rate of turn and speed.

Equations (38) and (39) will generate the curve containing all points reached by a continuous turn within the reachable set.

What remains to be determined is the slope of the line containing all possible points on the outer boundaries of all possible reachable subsets for a given change in heading and time delay for all possible turning rates *(Figure 10).*

From Equations (11) and (12)

$$\frac{x_o(\dot{\theta}_1) - x_o(\dot{\theta}_2)}{y_o(\dot{\theta}_1) - y_o(\dot{\theta}_2)} =$$

$$\frac{\sin\theta - \theta\cos\theta}{(1 - \cos\theta) - \theta\sin\theta} \quad (40)$$

and the slope is

$$\beta = \arctan\left[\frac{\sin\theta - \theta\cos\theta}{(1 - \cos\theta) - \theta\sin\theta}\right] \quad (41)$$

If Equation (32) is not very pleasant, Equation (41) is positively unpleasant. "Small angle" approximation yeilds

$$\beta \simeq \frac{2\theta}{3} \quad (42)$$

Obviously

$$\frac{\theta}{3} + \frac{2\theta}{3} = \theta \quad (43)$$

and one might suspect

$$\alpha + \beta = \theta, \quad (44)$$

which is indeed true as will be demonstrated.

First consider the case of a continuous turn, *Figure 11.*

$$AC = 2\ r \sin\ (\theta/2) \quad (45)$$

$$x = (2\ r \sin\ (\theta/2)) \cos \theta/2 \quad (46)$$

$$y = (2\ r \sin\ (\theta/2)) \sin \theta/2 \quad (47)$$

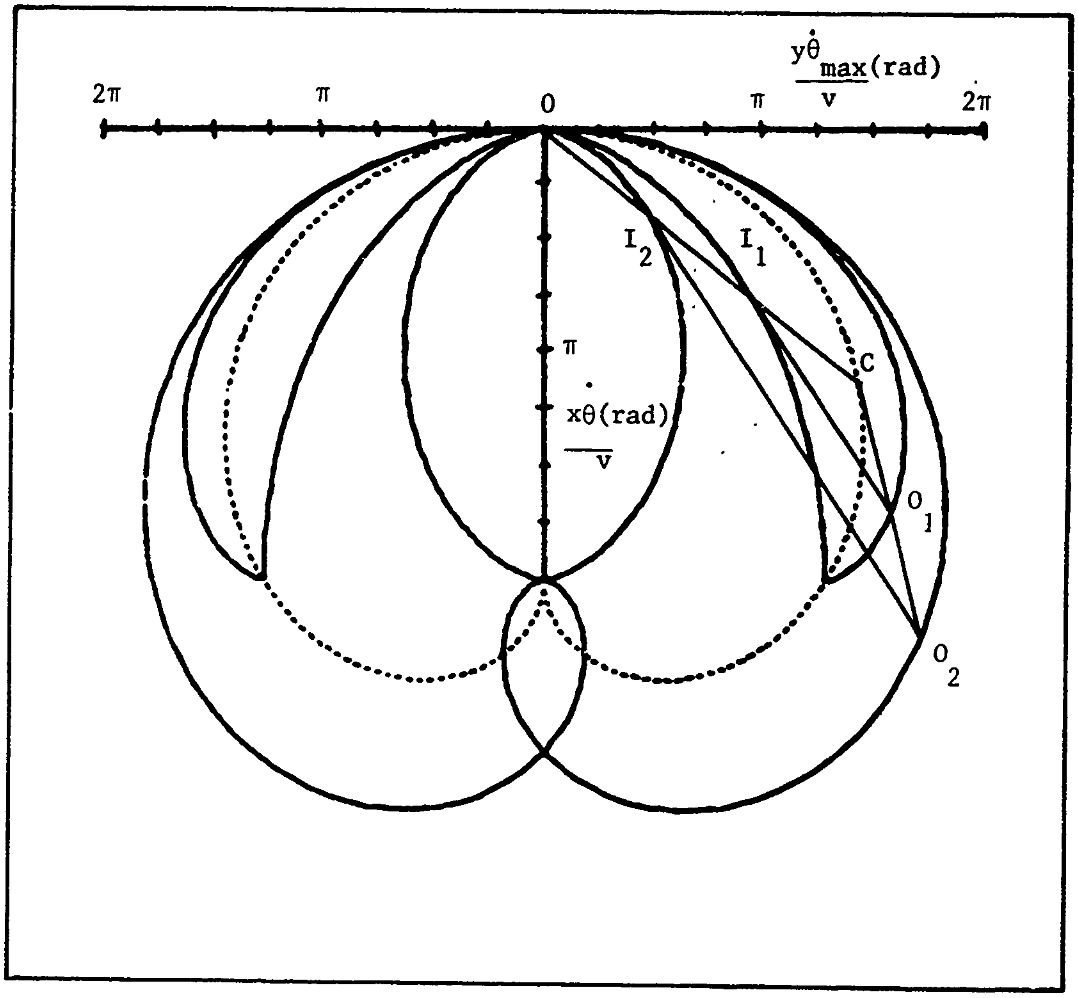

Figure 10. Reachable state and substate for 114° turn to the right.

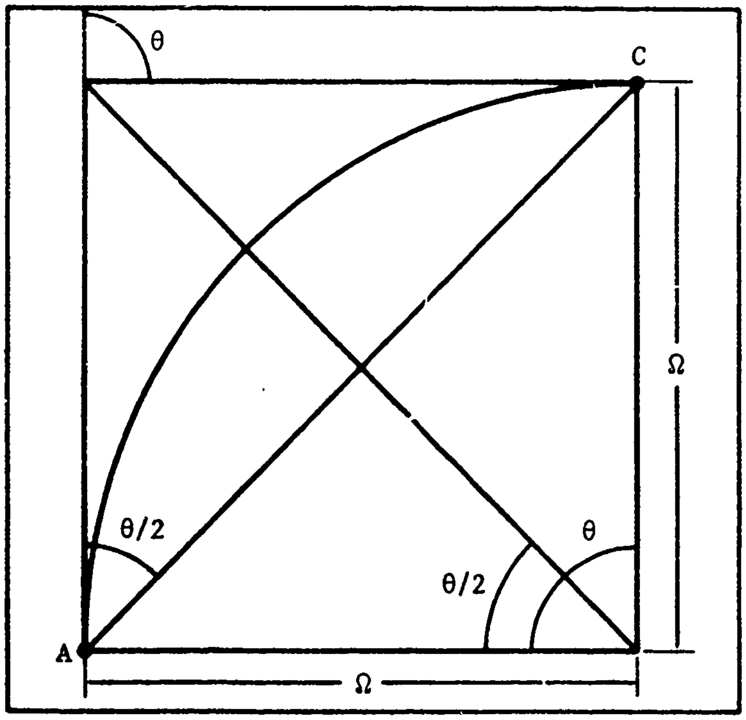

Figure 11. Continuous turn geometry.

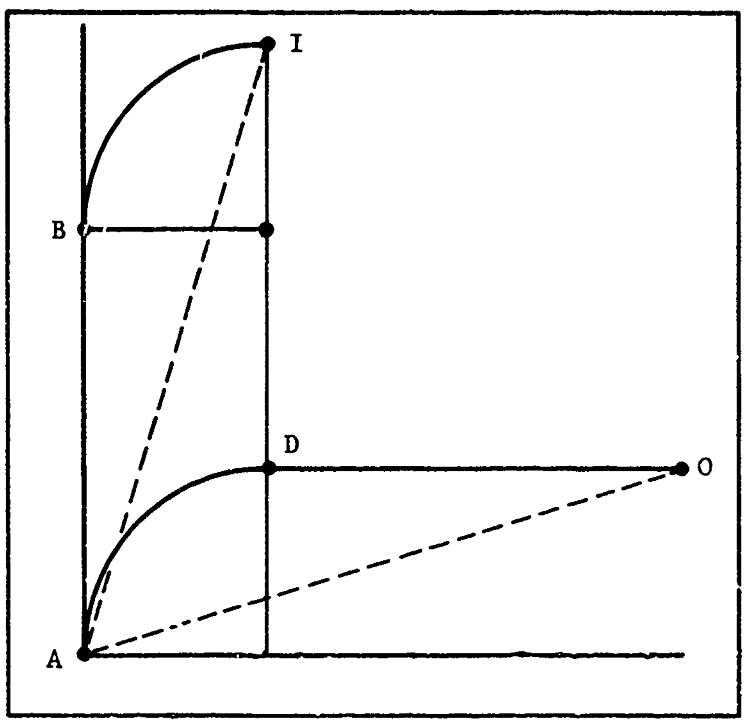

Figure 12. Initial and final turn geometry.

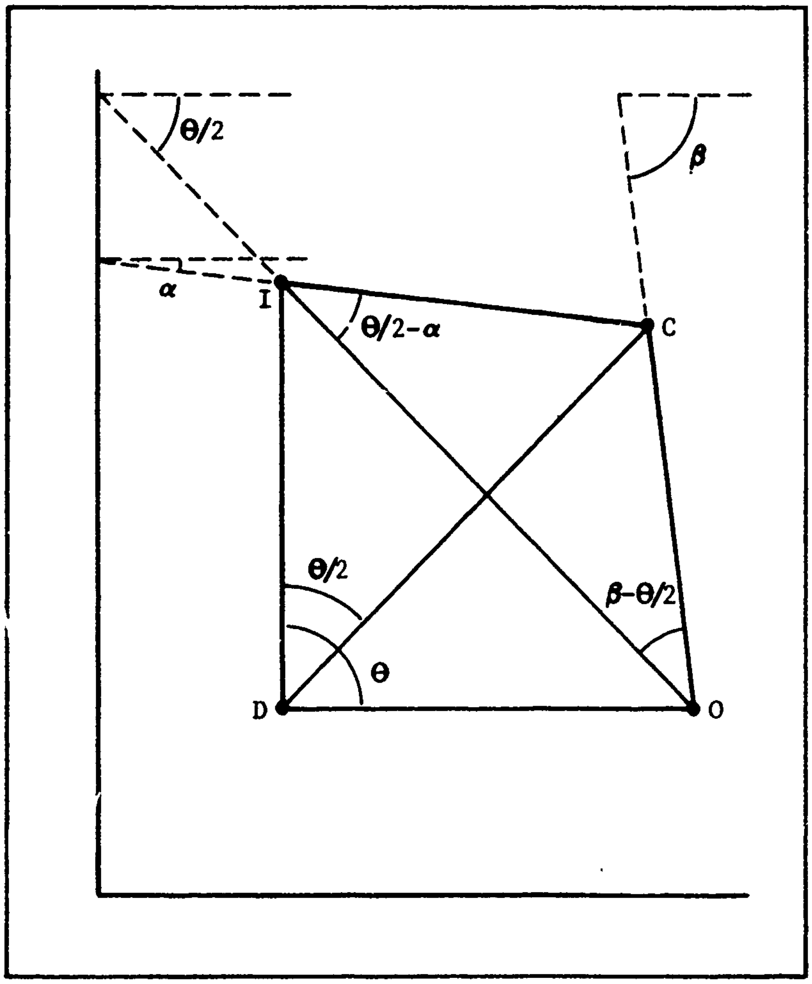

Figure 13. Turn geometries combined.

Also points on the inner and outer boundary that have the same change in heading are the same distance from the initial position *(Figure 12).*

The arc B I is equal to the arc A D and A B is equal D O, therefore A I is equal A O.

It follows that D I equals D O, therefore I M is equal M C, and I C is equal C O, ... , and, finally,

$$\frac{\theta}{2} - \alpha = \beta - \frac{\theta}{2} \qquad (48)$$

which simplifies to

$$\alpha + \beta = \theta . \qquad (49)$$

Equation (44) would be useful if one had computed α using Equation (32) since one may determine β using Equation (44) instead of Equation (41).

A word of caution; a change of heading, theta, could result from a "simple" turn or a turn of

$$\pm(2 n \pi \pm \theta) , n > o .$$

if time allows. For example, a change of heading of ninety degrees to the right could have resulted from a ninety degree turn to the right, a two hundred and seventy degree turn to the left, et cetera. For

$$\theta_{max} T < \pi,$$

there is no difficulty, which is also the practical limit of the small angle approximations given above. For a maximum turning rate of three degrees per second

$$T < 60s$$

will not result in difficulties. For longer delays all possibilities should be checked.

Since sufficient time must have elapsed for the change in heading to have occurred, one should first check to insure that

$$\theta_{max} = \dot{\theta}_{max} T$$

is not exceeded. This test will eliminate points which are obviously not members of the reachable state.

The reachable state for a one hundred and fourteen degree turn to the right and theta max of two pi is illustrated in *Figure 14.* For clarity the state for two hundred and forty-six degrees to the left, 360° - 114°, is not shown. How the reachable state may be implemented is as follows.

If the point P (X, Y) lies in the reachable state

$$\tan \alpha \leq \left(\frac{x_p - x_i}{y_p - y_i} \right) \leq \tan \gamma \qquad (50)$$

and

$$\frac{x_p - x_c}{y_p - y_c} \geq \tan \beta \qquad (51)$$

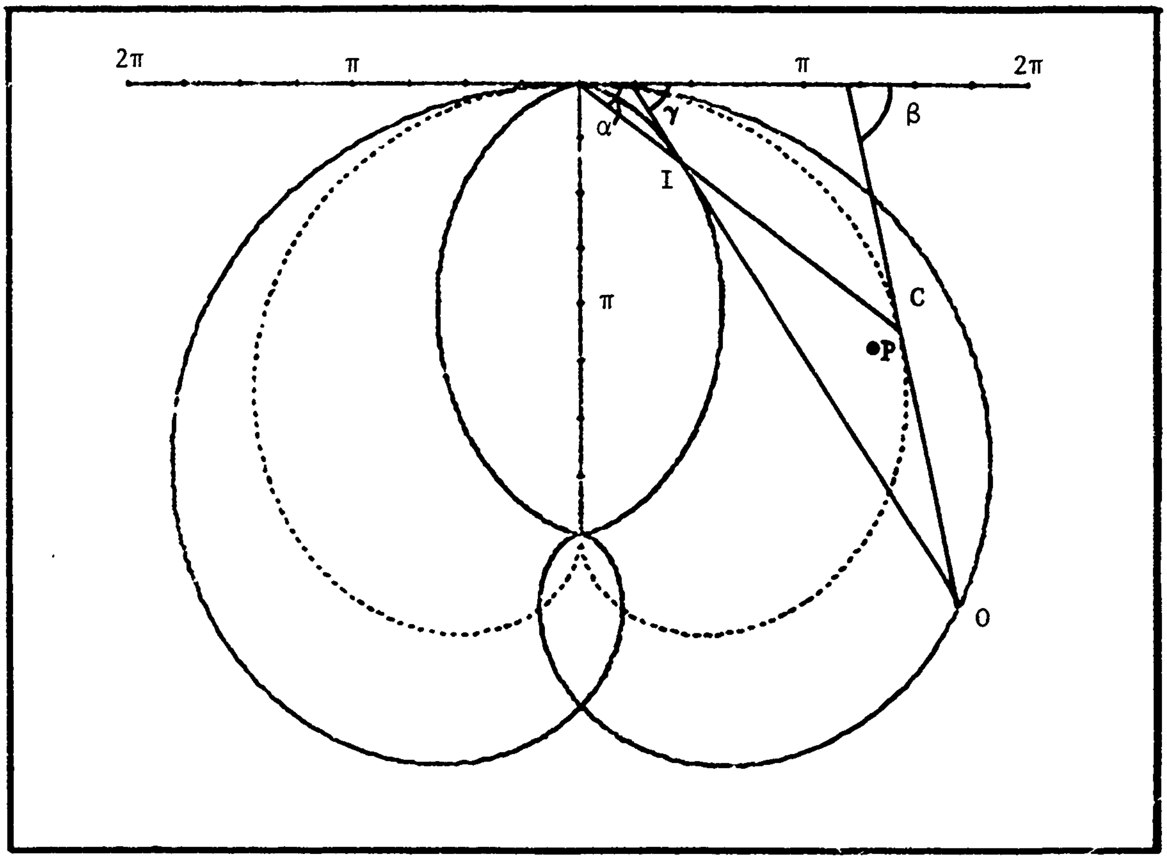

Figure 14. Reachable state fo: 114° turn to the right and a theta max of 2 π .

Substituting, Equation (50) and (51) become

$$\tan\left(\frac{\theta}{3}\right) \lesssim \left(\frac{x_p - \frac{v}{\theta}(\sin\theta - \theta)}{y_p - \frac{v}{\theta}(1 - \cos\theta)} \right) \lesssim \tan\left(\frac{\theta}{2}\right)$$

and (52)

$$\frac{x_p - \frac{vT}{\theta}(\sin\theta - \theta)}{y_p - \frac{vT}{\theta}(1 - \cos\theta)} \gtrsim \tan\left(\frac{2\theta}{3}\right) \qquad (53)$$

respectively.

4. RESULTS AND CONCLUSIONS

It was found advantageous to locate the origin of the reachable set at the linearly extrapolated position; the inner boundary is then common to all sets for a single turn. This then led to the reachable state where the change in heading is used to further reduce the allowable area in which the position may lie. The triangle of the reachable

state sweeps out the reachable set as one varies the change in heading from zero to the maximum possible for the time delay.

A point which should be made explicit is that the reachable sets/states not only answer the question as to what the aircraft could attain but how its present position could have been attained, that is, where it could have come from. Since one may be correlating track data which are members of sequences, those closest together in time, whether in the relative "past" or "future", should be checked.

A caveat, the only conclusion one may draw if a point fails the reachable state criteria, is that it does not correlate with an aircraft making a single horizontal turn at constant speed.

It appears that multiple turns could be addressed in a similar manner. The outer boundary would remain the same since it is a maximum [1]. However, for two opposing turns the inner boundary would be composed of epicycloids at the initial position. One would roll one circle of minimum radius around another on the other side of the linearly extrapolated flight path till the point reached the linearly extrapolated flight path and then repeat the process from the other side. Heuristically this appears to be the inner boundary for multiple turns but a proof has not been developed at this time. The reachable state for two opposing turns resembles the reachable set for a single turn with the radius of turn doubled [4]. The reachable set for double/multiple turns would have uses in determining acquisition cells. The equations are :

$$\frac{X_i \dot{\theta}}{v} = 2\sin\theta - \sin(\theta max - \pi - 2\theta) \tag{54}$$

$$\frac{Y_i \dot{\theta}}{v} = 1 - 2\cos\theta - \cos(\theta max - \pi - 2\theta) \tag{55}$$

The effects of uncertainties remain to be incorporated. The method suggested in Reference 1 would be rather pessimistic, but for determination of acquisition cells this simplicity may be acceptable.

The gedanken experiment is worth performing. The Kenner Spirograph® is readily available or consult Reference 3 for other methods — or design your own; a piece of string ...

The reachable sets/states for single turns are a possible means of correlation when the length of the time delays warrant. At the very least, they graphically illustrate the effects of time delays on track data. In addition to fixed wing aircraft there would also

be applications to tracking surface ships. Though motivated by the needs of tactical air defense and control, the potential for such techniques in commercial air traffic control is obvious.

REFERENCES

1. P. J. Wong and A. W. Korsak, *Reachable Sets For Tracking,* Operations Research, Volume 22, Baltimore, Maryland, pp. 497-509.

2. O. F. Forsyth, F. L. Crane, P. J. Wong and J. J. Penick, *AAW Track Handling: System Description and NTDS Detector/Tracker Model (U),* SRI-74-0694 (AD530950L), Stanford Research Institute, Menlo Park, California, July 1973, Confidential Report. (Appendix B, Reachable Sets and Correlation Contours, Unclassified).

3. H. M. Cundy and A. P. Rollett, *Mathematical Models*, Oxford at the Clarendon Press, London, 1961.

4. R. E. Dickson, US Army Missile Research and Development Command Laboratory Notebook Number 7066, Redstone Scientific Information Center, Redstone Arsenal, Alabama 35809.

DISTRIBUTION

	No of Copies
Defense Document Center Cameron Station Alexandria, VA 23144	12
Commander Naval Surface Weapons Center ATTN: C. E. Spooner, F-42 Dahlgren, VA 22448	1
Commander Naval Underwater Systems Center ATTN: E. J. Hilliard, Jr. Newport RI 02840	1
Rome Air Development Center RADC-OCTM ATTN: W. Wolf Griffiss AFB, Rome, NY 13441	1
Commandant US Army Air Defense School ATTN: Col. Wyatt Fort Bliss, TX 79916	1
Commander USAF Systems Command	
ESD/YWEH, Ltc. R. J Rodgers	1
ESD/DCYR, P. Shanley	1
L G. Hanscom AFB, MA 01731	
The MITRE Corporation ATTN: J. Leva P. O. Box 208 Bedford, MA 01730	1
Raytheon Company ATTN: F. Reynolds Hartwell Road Bedford, MA 01730	1

	No. of Copies
IBM ATTN: R. Churchill 150 Sparkman Drive Huntsville, AL 35805	1
Scientific Applications, Inc. ATTN: R. Jones Suite 800 2109 W. Clinton Avenue Huntsville, AL 35805	1
Stanford Research Institute ATTN: P. J. Wong Menlo Park, CA	1
Systems Control, Inc. ATTN: L. Meir/D. M. Salmon 260 Sheridan Avenue Palo Alto, CA 94306	1
DRCPM-MD-T-E, R Russell	10
DRCPM-TDS-RA-E	1
DRCPM-ROL-E	1
DRDMI-T, Dr Kobler	1
DRDMI-TBD	3
-TE	1
-TG	[illegible]
-TD	1
-TDW, R. Dickson	10
-CD	1
-CGA	1
-YR	1
-YS	1
-TI (Reference Copy)	1
(Record Copy)	1

☆U.S. GOVERNMENT PRINTING OFFICE: 1979–640-122/368